Total Differential 全微分(p900)

Differential of single variable functions:

$$y = f(x) \Rightarrow dy = f'(x)dx$$

Total Differential of multiple variable functions:

$$z = f(x,y) \Rightarrow dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Example 3: Find the total differential for each function.

a. $z = 2x\sin y - 3x^2y^2$

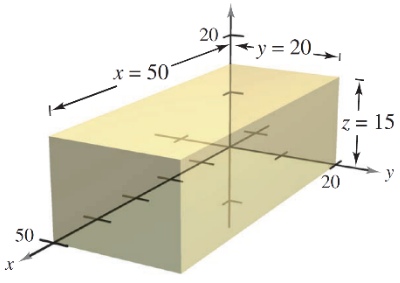
$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = (2\sin y - 6xy^2)dx + (2x\cos y - 6x^2y)dy$$

b. $w = x^2 + y^2 + z^2$

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz = 2xdx + 2ydy + 2zdz$$

Differentiability (p901)

Example 4: (Error Analysis) The possible error involved in measuring each dimension of a rectangular box is ± 0.1 millimeter. The dimensions of the box are $x = 500$ millimeter, $y = 200$ millimeter, and $z = 150$ millimeter, as shown in the right figure. Use dV to estimate the propagated error and the relative error in the calculated volume of the box.



Volume = xyz

Solution: $V = xyz$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz = yzdx + xzdy + xydz$$

$$dx = dy = dz = \pm 0.1$$

the propagated error is approximately

$$dV = 200 \times 150 \times (\pm 0.1) + 500 \times 150 \times (\pm 0.1) + 50 \times 20 \times (\pm 0.1)$$
$$= \pm 20500$$

$$V = 500 \times 200 \times 150 = 15\,000\,000 \text{ cubic millimeters}$$

the relative error $\Delta V/V$, is approximately

$$\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{20\,500}{15\,000\,000} \approx 0.14\%$$

Exercise: P906-32

32. The total resistance R (in ohms)of two resistors connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Approximate the change in R as R_1 is increased from 10 ohms to 10.5 ohms and R_2 is decreased from 15 ohms to 13 ohms.

$$R = \frac{R_1 + R_2}{R_1 + R_2}$$

$$\Delta R \approx dR = \frac{\partial R}{\partial R_1}dR_1 + \frac{\partial R}{\partial R_2}dR_2 = \frac{R_2^2}{(R_1+R_2)^2}\Delta R_1 + \frac{R_1^2}{(R_1+R_2)^2}\Delta R_2$$

$$R_1 = 10, R_2 = 15, \Delta R_1 = 0.5, \Delta R_1 = 0.5, \Delta R_2 = -2$$

$$\Delta R \approx \frac{15^2}{(10+15)^2}(0.5) + \frac{10^2}{(10+15)^2}(-2) = -0.14 \text{ ohm}$$

34. The centripetal acceleration of a particle moving in a circle is $a = v^2/r$ where v is the velocity and r is the radius of the circle. Approximate the maximum percent error in measuring the acceleration due to errors of 3% in v and 2% in r.

$$a = v^2/r$$

$$da = \frac{2v}{r}dv - \frac{v^2}{r^2}dr$$

$$\frac{da}{a} = 2\frac{dv}{v} - \frac{dr}{r} = 2(0.03) - (-0.02) = 8\%$$